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ABSTRACT

This paper is a reaction to a plenary address, "A Research Base Supporting Long Term Algebra Reform?" by James Kaput (SE 057 182). Three dimensions of algebra reform identified by Kaput (breadth, integration, and pedagogy) are discussed and contrasted with the draft version of the Algebra Document from the National Council of Teachers of Mathematics, which has categorized algebra into four themes: function, modeling, language or representation, and structure. Also critiqued are Kaput's three phases of reform (short, intermediate, and long term). (Contains eight references.) (MKR)

Algebra Reform, Research, and the Classroom: A Reaction to A research Base Supporting Long Term Algebra Reform

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ALGEBRA REFORM, RESEARCH, AND THE CLASSROOM: A REACTION TO A RESEARCH BASE SUPPORTING LONG TERM ALGEBRA REFORM

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The Aspects of Algebra

The five aspects of algebra, as generalizing, abstracting, and representing; manipulation of formal objects; as a study of structures; as a language; as a study of functions, relations and joint variation; as a means of controlling physical or cybernetic events, described by Kaput in his paper seem to be consistent with the organizing themes postulated in the draft version of the Algebra Document from the National Council of Teachers of Mathematics (NCTM, in preparation). Both documents begin from the premise that algebra has different faces and is interpreted very differently by different people. The NCTM (in preparation) Algebra Document uses as exemplars four organizing themes: function, modeling, language or representation, and structure. The themes in the NCTM document, which can be almost directly mapped to Kaput's aspects, are not meant to be inclusive but to suggest that there are a variety of ways in which people construct a view of algebra. An important point in the NCTM (in preparation) Algebra Document is that no one theme in itself seems to be sufficient to give students a complete picture of what it means to know and be able to do algebra. A collection of themes provides a teacher with multiple entry points into the ways children think about algebra. It is interesting to note that from the perspective of modeling, the two forms of a linear equation, $y = a + bx$ and $ax + by = c$, seem to students to be unrelated because the physical situations that generate each model are too different. From an organizing theme of structure, however, it is easy to demonstrate that the two are equivalent equations. Whether you think of themes or aspects, the question becomes, How can a curricular sequence be designed to incorporate different perspectives in a way that will give students a coherent and useful understanding of algebra?

The NCTM (in preparation) Algebra Document postulates that thinking and reasoning in algebra must be *about something*, and so suggests that the themes build student understanding through activities embedded in contextual settings, such as growth and change, data, and uncertainty; number, size and shape; or patterns and regularity. These settings provide an opportunity for students to make connections bet veen algebra and other disciplines and to see how algebra helps make sense of patterns in areas such as biology or economics or can be used to describe relations between geometric figures. Kaput's integration dimension might be linked to this notion of setting.

The third dimension, pedagogy is, as Kaput indicates, critical in how students come to perceive algebra, not as a set of magic tricks that can be used effectively only by someone who has the "math gene," but rather as a discipline useful to everyone, that has a logic and beauty of its own. Instruction should move toward an active exploration of algebraic ideas and exploit technology as a vehicle to

enable students to build an understanding of algebra and what it can do. Kaput's caution that "it is possible to achieve surface forms of valued pedagogies while failing entirely to engage students with significant mathematics" needs to be taken seriously. Not all rich investigations are worth doing. Is it really useful to fit a cubic to a piece of a curve generated by data from a CBL? Not all algebra can be constructed through rich investigations; at some point, students do need to have some facility with algebraic ideas (The question of which ideas and how much facility is not at all clear, however.). It is also not the case that one form of delivery should be the dominant form. While discourse, where ideas and interpretations are shared, discussed, and dissected, is a vital part of understanding for most people, real learning also depends on individual reflection and internalization.

Current Practice/Short Term Reform

Current practice has major implications for thinking about future directions. The baseline described by Kaput, essentially confined to two courses, broken into small categories of activities largely consisting of symbol manipulation and "story" problems, however, has a long history in the United States. In the 1909 text, *First Course in Algebra,* designed for a fourteen year old, the authors state in the preface, "A serious effort has been made to utilize the valuable suggestions in which the widespread discussion of the teaching of algebra for the last ten years has been fruitful....The material itself has been selected with the intention of affording the student ample drill in the elementary technic of algebra and a commensurate development of his reasoning power....Especial care has been used in the selection of the exercises in equations, the object being to have as great a variety as possible and yet to give only equations whose roots can be verified with a reasonable amount of labor" (Hawkes, Luby & Touton, 1909, pp. iii-iv). They describe the presence of frequent review exercises in factoring so students will "acquire in the shortest possible time a secure grasp of forms and methods and the careful blend of problem situations with technical work to avoid spending long periods of time on mere technic" (Hawkes, Luby & Touton, 1909, pp. iv). Typical sets of exercises include: 36 distance-time-rate problems, 80 factoring problems, and a chapter review with 66 problems. There are two essential differences between this book and the pre reform texts of the 90s: The content and exercises have become divided into two parts over the years, Algebra I and Algebra II, and the manipulation processes are based on axioms and rules. The historical notes are also different and quite interesting!

Technology possesses the power to change this at-least-one-hundred-year-old picture and to do so in dramatic ways. The graphing calculator has already had a major impact on the secondary mathematics curriculum, changing the focus of instruction and expectations about what is important for students to know. Each year, literally thousands of teachers attend calculator workshops where they learn to use a calculator to teach algebra, trigonometry and precalculus. These calculators are making the questions (solve, simplify, and factor) trivial which teachers used to teach students to answer from 1909 to 1995, and teachers are not sure what

new questions should take their place. In 1989 several of the chapters in *Research Issues in the Learning and Teaching of Algebra* (Wagner & Kieran, 1989) raised the issue of the need for research on the impact of technology on the algebra curriculum. Yet six years later there still seem to be very few studies available that can begin to enlighten those in the classroom about the directions they should be taking. Teachers are making changes, and text books are incorporating the changes. The question is, Are these changes contributing to the algebraic (and mathematical) knowledge of students in ways that are important? Is a coherent picture of algebra the driving force? Some of the changes might be seen as driven by what is possible or what seems to be motivating, without care given to what is important in overall understanding. The lag between research and curricular change is placing the mathematics community in a reactive position, rather than a proactive one.

Further, the development of technology promises to continue at an ever increasing rate. Technology should make a difference; it can move students beyond what is currently possible in school algebra and can do so in new ways. Yet, those working on curriculum design must think deeply about a curriculum that capitalizes on technology, but is not out of date by the time it is ready for print. They must also take care that the emphasis on technology does not create a new morass of symbols and procedures.

As Kaput indicates, algebra has already begun to change for a large number of schools and teachers. Many students now begin algebra in grade 8 or earlier. The content of the early algebra courses varies from what used to be considered prealgebra to a traditional ninth grade course to a version of reform algebra. There are currently many different "reform" movements underway, and some of these have been in place long enough to begin to produce results (although what results are desirable is not clear). Are these current changes making a difference and what effect will they have on the long term reform efforts? As an example, some of those who have embraced the graphing calculator have placed functions (explicitly defined) at the heart of algebra, and no longer pay much attention to equations generated to provide an algebraic representation of a plane or an algebraic statement of condition. In fact, there is a trend towards replacing traditional story problems with new versions; out go upstream, downstream, work, money, integer, and mixture problems, and in come the taxi cab, interest, and tossing a ball.

While there are inherent problems with the implementation of algebra for all, from poorly prepared students and enormous failure rates to inappropriate texts, there are also some gains. For some cities, such as Milwaukee, that mandated algebra for all, even though the failure rate is high, more students are passing algebra than before the mandate when only selected students took the course.

Kaput's case study of change regarding a Presidential Award winner's use of graphing calculators and the failure of this approach to produce students who understood function is an inappropriate one to use as evidence. Consider first that change without understanding what is important will not succeed. Consider also, the tenuous nature of what it means to understand function. Students have always had difficulty understanding function, and teachers and texts have, probably unknowingly, contributed to the problem. "These students perceptions of what con-

stitutes a function could be dismissed as just so many misconceptions, that is, errors in their conceptualizations....Indeed, all of these misconceptions are due to *an overgeneralization of the initial examples used in the introduction of the function concept*. These overgeneralizations, which occur naturally, become hurdles in the construction of the more global notion of function" (Herscovics, 1989, p. 80, emphasis added). Malik (1980) in a historical review traces the definition of function since Euler and indicates that each new definition of function was the subject of heated debate among the community (and most teachers probably do not know there are different definitions). Herscovics conclusion is that "learner's difficulties with functions are reflections of the history of the evolution of the concept." The bottom line is, a) did the teacher have sufficient knowledge and understanding of function and how to think about it (Many current secondary teachers were taught the set theoretic definition of function, one derided by Freudenthal (1973) and others), and b) what has research provided to help the teacher understand the cognitive difficulties students face in trying to understand the concept? New tools that do the same thing will not solve old problems. Our challenge is to determine how to make these new tools do new things that will solve these problems and, as part of that process, to provide a channel for using the knowledge gained from research to inform teaching.

Phase Two: Intermediate Reform/Backlash

The major curriculum projects are suggesting changes in the way algebra is construed as a school subject and in the way it is presented to students. Information about the success of these projects in building students' understanding of algebra so far seems to be primarily anecdotal, not surprising since, at the middle school level, the materials are just being completed, and few students have experienced a complete program. The high school materials are still in the developmental stage. Although examples of student work from the University of Wisconsin middle school project, *Mathematics in Context*, do support the success of the principle of "progressive formalization" described by van Reeuwijk (in preparation), long term research is needed.

As reform moves into this second phase, the problems are exacerbated by the issues raised above regarding the status quo. The change will not come easily and, in many places, those who advocate reform are already experiencing severe criticism. The tradition of algebra is firmly ensconced in the minds of everyone who had any experience, whether successful or unsuccessful, with algebra. Many teachers themselves are reluctant to abandon what they have done "successfully" for years. They "know" what their students should learn in an algebra course. Those who are willing to change are not sure to what they should change nor how to integrate this with the rest of the mathematics they are teaching. Algebra has been a filter, and parents are convinced that their children should learn what they themselves learned (or didn't learn) in order to make it past the filter. They perceive that changes in algebra will change the rules for success, and they are not about to do this lightly. The results of a survey for the NCTM (in preparation) Algebra

Document indicated a surprising number of educators who do not think everyone can learn algebra. Many mathematicians and scientists are concerned that a new algebra, driven in many cases by technology, is essentially providing a watered down curriculum that will produce students with little if any mathematical knowledge. Finally, many parents do not want their children to be part of an "experiment." The foundation for algebra reform should have some roots in research, and an analysis of such research, in language accessible to teachers and parents, should be used to provide support for the direction of reform.

Phase Three/Long Term Reform

The NCTM (in preparation) Algebra Document is based on three assumptions: A reconceptualization of algebra is needed; all students can learn algebra and should have the opportunity to do so; and algebra should be taught throughout the K-12 curriculum. The long term reform advocated by Kaput implicitly builds from these same three assumptions. Success depends on major shifts by researchers, mathematicians, mathematics educators, and teachers working through the issues together. Researchers (and mathematicians) focus on whether the value of the quadratic formula lies in its ability to complete the analysis of quadratics (Thorpe, 1989) or because its development depends on a very important mathematical tool, completing the square. Teachers, who once taught students just to use the formula, have to make a major shift—what does it mean to complete the analysis of a quadratic; what does a mathematical tool such as completing the square buy for you? Such shifts will help teachers and students focus on the algebraic concepts that are important, not the output of the algorithm. The background and preservice training of K-12 teachers is currently inadequate for such thinking. (Ask any ninth grade algebra teacher to suggest a coherent sequence for the topics in algebra.) Mathematicians need to recognize that the community will be well served if some of the traditional K-12 content is replaced by concepts from discrete topics, from statistics or linear algebra.

Some of the shifts brought by the short and intermediate phase of reform will have a major impact on long term efforts. In some cases manipulatives have begun a life of their own; elementary teachers are reluctant to even think about teaching "algebra;" some new curricula carry reform directions to an extreme (no factoring, no practice); and "tracking" issues are critical. In every case, research can help educators make decisions. Kaput cites many examples of current research efforts. Unfortunately, the channels for communicating the results of that research to those in the classroom and for promoting dialogue among the mathematics community are not in place. Our task is to establish these channels as well as investigate algebraic teaching and learning if we are to have effective long term reform.

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